A Low Complexity and Robust Frequency Offset Estimation Algorithm for OFDM-Based WLAN Systems*

Sanghun Kim[1], Seokho Yoon[1], Hyoung-Kee Choi[1], and Sun Yong Kim[2]

[1] School of Information and Communication Engineering, Sungkyunkwan University, 300 Chunchun-dong, Jangan-gu, Suwon, Kyunggi-do, 440-746, Korea
{ksh7150, syoon, hkchoi}@ece.skku.ac.kr
[2] Department of Electronics Engineering, Konkuk University, 1 Hwayang-dong, Gwangjin-gu, Seoul 143-701, Korea
kimsy@konkuk.ac.kr

Abstract. This paper addresses the frequency offset estimation problem in the presence of the timing error for OFDM-based WLAN systems. When the timing error exists, the correlation value used for the frequency offset estimation could be reduced significantly due to the timing error, resulting in considerable degradation in estimation performance. In this paper, using the coherence phase bandwidth (CPB) and a threshold, a novel frequency offset estimation algorithm is proposed and based on which, an efficient timing error estimation algorithm is also proposed for the re-estimation of the frequency offset.

1 Introduction

Due to its high spectral efficiency and immunity to multipath fading and impulsive noise, orthogonal frequency division multiplexing (OFDM) technology has been widely adopted as a modulation scheme in wireless local area network (WLAN) standards, such as institute of electrical and electronics engineers (IEEE) 802.11a, high performance local area network type 2 (HiperLAN/2), and mobile multimedia access communication (MMAC), [1]. The OFDM technology, however, is very sensitive to frequency offset caused by Doppler effect and the mismatch of the oscillators in the transmitter and receiver [2]. Therefore, exact estimation of the frequency offset is one of the important technical issues in OFDM-based systems including WLAN [3].

Although a number of frequency offset estimation algorithms have been proposed [4]-[7], most of the investigations have been discussed under the assumption of no timing error. Practically, it is very difficult to determine the OFDM symbol arrival time at the receiver prior to the frequency offset estimation, because the signal-to-noise ratio (SNR) before the frequency offset estimation is

* This research was supported by grant No. R01-2004-000-10690-0 from the Basic Research Program of the Korea Science & Engineering Foundation. Dr. Yoon is the corresponding author.

V.N. Alexandrov et al. (Eds.): ICCS 2006, Part II, LNCS 3992, pp. 961–968, 2006.
© Springer-Verlag Berlin Heidelberg 2006

very low. Therefore, it is not reasonable to assume that the timing error does not exist during the frequency offset estimation process. An efficient frequency offset estimation algorithm that is robust to the timing error was presented by Bang [8]. Bang's algorithm has good performance even though a timing error exists; however, the complexity in implementation rapidly increases as the frequency offset range increases.

In this paper, we propose a novel frequency offset estimation algorithm using the coherence phase bandwidth (CPB) and a threshold. The proposed algorithm has similar performance to that of Bang' algorithm in the presence of the timing error, yet has even lower computational complexity when compared with Bang's algorithm. We also address the problem of estimating the timing error, using the OFDM symbol compensated by the proposed frequency offset estimation algorithm.

2 The Effect of Symbol Timing Error on Frequency Offset Estimation

An OFDM symbol is generated by the inverse fast Fourier transform (IFFT) and expressed as, for $n = 0, 1, 2, \cdots, N-1$,

$$s_n = \frac{1}{\sqrt{N}} \sum_{l=0}^{N-1} S_l e^{j2\pi nl/N}, \tag{1}$$

where S_l is a phase shift keying (PSK) or quadrature amplitude modulation (QAM) data symbol transmitted through the lth subcarrier and N is the size of the IFFT. In the presence of the timing error, the received signal r_n can be expressed as

$$r_n = s_{n-\delta} e^{j2\pi\epsilon(n-\delta)/N} + w_n, \tag{2}$$

where ϵ and δ represent the frequency offset and the timing error, which are normalized to the subcarrier spacing $1/N$, respectively and w_n is the zero-mean complex additive white Gaussian noise (AWGN).

To recover the data symbol, the received signal is first demodulated using the fast Fourier transform (FFT) operation. The kth FFT output R_k is then given by, for $k = 0, 1, 2, \cdots, N-1$,

$$R_k = S_{k-\epsilon} e^{-j2\pi\delta(k-\epsilon)/N} + W_k, \tag{3}$$

where W_k is the FFT output of w_n. We consider the frequency offset estimation using the preamble, as in [4]-[8]. Thus, an estimate $\hat{\epsilon}$ of the frequency offset ϵ can be obtained as

$$\hat{\epsilon} = \arg\max_{d} \left\{ \left| \sum_{k=0}^{N-1} Z_k^* R_{(k+d)_N} \right| \right\}. \tag{4}$$

where Z_k is the known preamble, $*$ is the complex conjugate operator, d is the amount of cyclic shift, and $(\cdot)_N$ is the modulo-N operator.

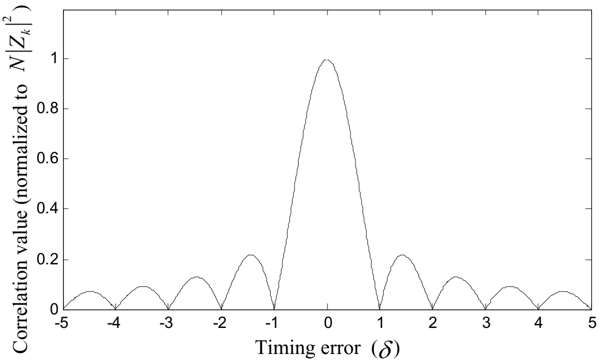

Fig. 1. The correlation value plotted as a function of timing error δ when $d = \epsilon$ and $N = 1024$.

When $d = \epsilon$ (i.e., when ϵ is correctly estimated), the correlation value (normalized to $N\,|Z_k|^2$) in (4) becomes $\left| e^{\frac{-j\pi\delta(N-1)}{N}} \frac{\sin(\pi\delta)}{\sin(\pi\delta/N)} \right|$ and is plotted as a function of the timing error δ, as shown in Fig. 1. From Fig. 1, we can clearly observe that the correlation value used in the frequency offset estimation is very sensitive to the variation of the timing error, which implies that the correlation value could be reduced significantly due to the timing error even if the frequency offset is correctly estimated, resulting in considerable degradation in estimation performance (note that a large correlation value when $d = \epsilon$, is essential for the detection of the correct frequency offset estimate). This is the very effect of the timing error on the frequency offset estimation.

3 Proposed Algorithm

In Bang's algorithm, the correlation is performed in parallel for all possible values of d. As the spacing between the consecutive values of d becomes smaller, therefore, the performance becomes more robust to the timing error variation; however, the computational complexity increases rapidly. In this paper, we propose an efficient algorithm that has similar performance, but less complexity for a given value of d, when compared with Bang's algorithm.

3.1 Frequency Offset Estimation

To alleviate the effect of the timing error on the frequency offset estimation, we consider the CPB within which the correlation value increases monotonically. The CPB can be expressed as [8]

$$\mathrm{CPB} = \frac{N}{2\delta^t}, \tag{5}$$

where δ^t is a maximum tolerated value of the timing error δ. Using (4) and (5), we can calculate the correlation value C for the frequency offset estimation as

$$C = \sum_{m=0}^{K-1} \left| \sum_{k=0}^{\mathrm{CPB}-1} Z^*_{k+m\mathrm{CPB}} R_{(k+m\mathrm{CPB}+d)_N} \right| , \tag{6}$$

where $K = N/\mathrm{CPB}$.

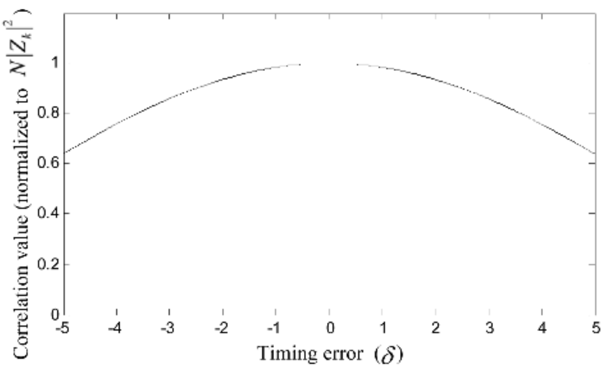

Fig. 2. The correlation value plotted as a function of the timing error δ when $d = \epsilon$, $\delta^t = 5$, and $N = 1024$.

Fig. 2 shows the correlation value plotted as a function of the timing error δ when $d = \epsilon$, $\delta^t = 5$, and $N = 1024$. It can be observed that the correlation value is relatively constant under the timing error variation. In [8], the correlation value C is calculated in parallel for all possible values of d and $\hat{\epsilon}$ is chosen as the d corresponding to the maximum correlation value. In the proposed algorithm, on the other hand, the correlation value C is first calculated for an initial d and then compared with a given threshold. If the correlation value exceeds the threshold, the d is decided to be the correct estimate of ϵ. Otherwise, the FFT output is cyclically shifted by d and the above procedure is repeated. The operation of the proposed algorithm is described in Fig. 3. The threshold η is obtained as follows.

From (6), correlation value α over one CPB block is

$$\alpha = \left| \sum_{k=0}^{\mathrm{CPB}-1} Z^*_k R_{(k+d)_N} \right| . \tag{7}$$

If we assume $d = \epsilon$ and ignore AWGN,

$$\alpha = |Z_k|^2 \left| \sum_{k=0}^{\mathrm{CPB}-1} e^{-j2\pi\delta k/N} \right| . \tag{8}$$

It is easy to see that α has its minimum value $\alpha_{\min}$ when $\delta = \delta^t$. Thus,

$$\alpha_{\min} = |Z_k|^2 \left| 1 - j \cot\left(\frac{\pi\delta^t}{N}\right) \right| \tag{9}$$

and the minimum value $C_{\min}$ of C can be obtained as

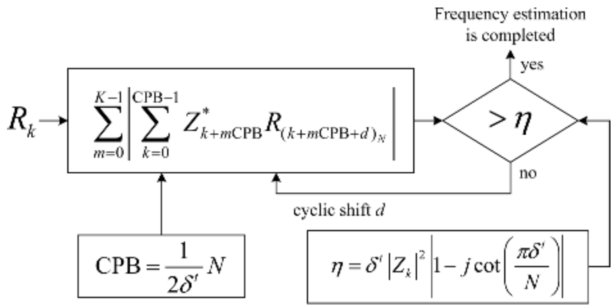

Fig. 3. Block diagram of the proposed frequency offset estimation algorithm

$$C_{\min} = 2\delta^t \left|Z_k\right|^2 \left|1 - j\cot\left(\frac{\pi\delta^t}{N}\right)\right|. \tag{10}$$

Finally, the threshold η is chosen such that

$$\eta = C_{\min}/2 = \delta^t \left|Z_k\right|^2 \left|1 - j\cot\left(\frac{\pi\delta^t}{N}\right)\right|. \tag{11}$$

As a threshold becomes smaller, the detection probability for the correct estimate of ϵ and the false alarm probability increase, and vice versa. Thus, the threshold chosen as one half of $C_{\min}$ is one of the reasonable choices in view of the detection and false alarm probabilities, which decide the overall system performance.

3.2 Timing Error Estimation

The frequency offset is often time varying and, thus, the re-estimation for the frequency offset is required. In that case, it would be very useful if an estimate of the timing error is available, although the proposed algorithm is robust to the timing error. So, we address the problem of estimating the timing error, using the FFT output (i.e., OFDM symbol in the frequency domain) compensated by the proposed frequency offset estimation algorithm.

Assuming the frequency offset is perfectly estimated, we have the kth FFT output R_k such that

$$R_k = H_k Z_k e^{j2\pi\delta k/N}. \tag{12}$$

where H_k represents the frequency response of the channel. The argument Φ_k of $Z_k^* R_k$ is then given by

$$\Phi_k = \angle\left(Z_k^* R_k\right) = 2\pi\delta\frac{k}{N} + \angle\left(H_k\right), \tag{13}$$

where $\angle$ denotes the argument operator of a complex number. As a result, if the channel does not change significantly during one OFDM symbol duration, the timing error can be estimated as follows.

$$\hat{\delta} = \frac{N}{2\pi}E\left\{\Delta_k\right\}, \quad \text{for } k = 1, 2, \cdots, N-1, \tag{14}$$

where $|\Delta_k| = |\Phi_k - \Phi_{k-1}| \leq 2\pi\delta^t/N$ and $E\{\cdot\}$ denotes the expectation and is taken to reduce the variance of the estimator.

4 Performance Comparison Results

In this section, the proposed frequency offset estimation algorithm is compared with Bang's algorithm in terms of estimation accuracy and complexity in implementation. Also, the performance of the proposed timing error estimation algorithm is presented. We consider the following parameters: $N = 1024$ and 2048, $\epsilon \in [0, 500]$, and $\delta^t = 16$. Two channel models are used: AWGN and multipath channels, the parameters of which are shown in Table 1. The time delays and amplitude attenuations of the second, third, and fourth paths are relative to those of the first path. For reference, the conventional frequency offset algorithm proposed by Nogami [4], which uses the assumption of no timing error, is also included in the comparison results.

Table 1. The channel models

Path	**First**	**Second**	**Third**	**Fourth**
Time delay (sample)	0	5	10	15
Amplitude attenuation (dB)	0	4	8	12
Phase	Distributed uniformly over $[0, 2\pi)$			
AWGN	10 dB			

Table 2 shows the complexity of Nogami's, Bang's, and the proposed algorithms. We can see from Table 2, for $L \gg 1$, where L denotes the frequency offset range, the proposed algorithm has about a half computational complexity when compared with Nogami's and Bang's algorithms. Another important result is that the proposed algorithm does not require any memory for correlation value, unlike Nogami's and Bang's algorithms.

Table 2. Complexity comparison for Nogami's, Bang's, and the proposed algorithms

Algorithm	**Number of complex multiplication**	**Number of comparison operation**	**Size of memory for correlation value**
Nogami's Algorithm	LN	$L-1$	L
Bang's Algorithm	LN	$L-1$	L
Proposed Algorithm	$\frac{L+1}{2}N$	$\frac{L+1}{2}$	-

Tables 3 and 4 show the frequency offset estimation accuracy of Nogami's, Bang's, and the proposed algorithms in AWGN and multipath channels, respectively. From the tables, it is seen that Bang's and the proposed algorithms are quite robust to the timing error and dramatically outperform Nogami's algorithm.

Table 3. Accuracy comparison of the frequency offset estimation algorithms in AWGN channel

Timing error (sample)	Accuracy (%)					
	Number of subcarrier (N): 1024			Number of subcarrier (N): 2048		
	Nogami's Algorithm	**Bang's Algorithm**	**Proposed Algorithm**	**Nogami's Algorithm**	**Bang's Algorithm**	**Proposed Algorithm**
0	100	100	100	100	100	100
1	0	100	100	0	100	100
2	0	100	100	0	100	100
5	0	100	100	0	100	100

Table 4. Accuracy comparison of the frequency offset estimation algorithms in multi-path channel

Timing error (sample)	Accuracy (%)					
	Number of subcarrier (N): 1024			Number of subcarrier (N): 2048		
	Nogami's Algorithm	**Bang's Algorithm**	**Proposed Algorithm**	**Nogami's Algorithm**	**Bang's Algorithm**	**Proposed Algorithm**
0	100	100	100	100	100	100
1	0	100	100	0	100	100
2	0	100	100	0	100	100
5	0	100	100	0	100	100

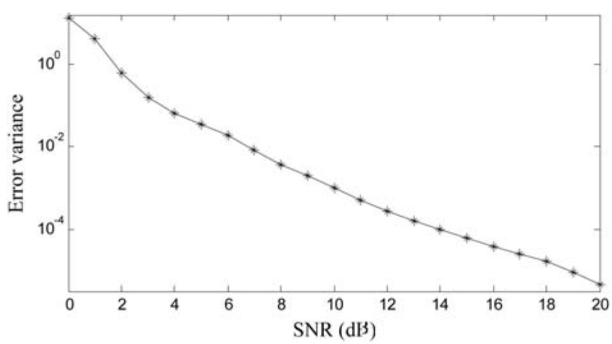

Fig. 4. Error variance of the proposed timing error estimation algorithm in AWGN channel

Figs. 4 and 5 show the error variance of the proposed timing error estimation algorithm as a function of SNR in AWGN and multipath channels, respectively. From the figures, we can observe that the error variance decreases rapidly as the SNR increases: specifically, the error variance becomes about 10^{-3} at SNR=10 dB, which is acceptable in general wireless applications and, thus, the proposed estimator should be able to be employed for the re-estimation of the frequency offset.

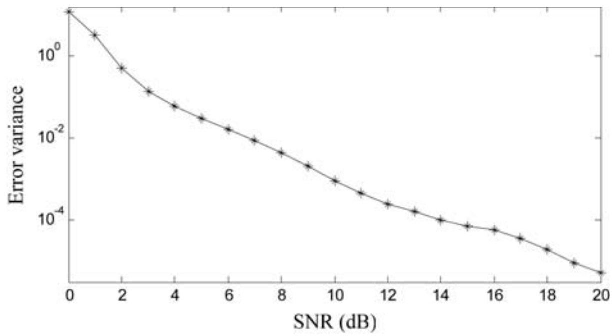

Fig. 5. Error variance of the proposed timing error estimation algorithm in multipath channel

5 Conclusion

We have first investigated the effect of the timing error on the frequency offset estimation in OFDM-based WLAN systems. Using the CPB and a threshold, then, we have proposed a low complexity and robust frequency offset estimation algorithm in the presence of the timing error. Based on which, an efficient timing error estimation algorithm has also been proposed for the re-estimation of the frequency offset. The comparison results have shown that the proposed frequency offset estimation algorithm dramatically outperforms Nogami's algorithm and performs similarly to Bang's algorithm in the presence of the timing error, with about a half complexity. It has also been shown that the proposed timing error estimator gives a reliable estimate of the timing error and, thus, should be useful for the re-estimation of the frequency offset.

References

1. N. Prasad and A. Prasad, *WLAN Systems and Wireless IP for Next Generation Communications*. Boston, MA: Artech House, 2002.
2. T. Pollet and M. Peeters, "Synchronization with DMT modulation," *IEEE Commun. Mag.*, vol. 37, pp. 80-86, Apr. 1999.
3. R. V. Nee and R. Prasad, *OFDM for Wireless Multimedia Communications*. London, England: Artech House, 2000.
4. H. Nogami and T. Nagashima, "A frequency and timing period acquisition technique for OFDM systems," *in Proc. IEEE PIRMC*, Toronto, Canada, pp. 1010-1015, Sep. 1995.
5. B. Y. Prasetyo, F. Said, and A. H. Aghvami, "Fast burst synchronisation technique for OFDM-WLAN systems," *IEE Proceedings: Commun.*, vol. 147, pp. 292-298, Oct. 2000.
6. J. Li, G. Liu, and G. B. Giannakis, "Carrier frequency offset estimation for OFDM-based WLANs," *IEEE Signal Process. Lett.*, vol. 8, pp. 80-82, Mar. 2001.
7. K. Fazel and S. Kaiser, *Multi-Carrier and Spread Spectrum Systems*. West Sussex, England: John Wiley and Sons, 2003.
8. K. Bang, N. Cho, H. Jun, K. Kim, H. Park, and D. Hong, "A coarse frequency offset estimation in an OFDM system using the concept of the coherence phase bandwidth," *IEEE Trans. Commun.*, vol. 49, pp. 1320-1324, Aug. 2001.